RIGOROUS BAYESIAN APPROACH TO DEVELOP MAXIMUM MAGNITUDES DISTRIBUTIONS FOR PSHA COMPUTATIONS

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***Abstract**: Probabilistic Seismic Hazard Assessment (PSHA) has the goal to evaluate annual frequencies of exceeding a given ground motion Intensity Measure. One important parameter in the seismicity models used in PSHA is the maximum magnitude that can be expected in a region. This is true in particular for nuclear safety applications where higher return periods need to be considered. This paper describes an improved approach for the estimation of the maximum magnitude in the truncated GR law by means of a Bayesian approach involving extreme value statistics and accounting for uncertainties. The method to constructs the likelihood function based on the distribution of extremes of the truncated GR law and is an improvement of former developments by EPRI and further promoted by USNRC. In the proposed method, only the completeness period of the maximum observed earthquake is required, so that there is no need to determine and use the exact completeness periods for magnitude bins of smaller events and to introduce the associated uncertainties. This makes the approach easy to implement and to apply. Eventually we highlight possible impact on probabilistic hazard assessment results.*

1. Introduction

Probabilistic Seismic Hazard Assessment (PSHA) has the goal to evaluate annual frequencies of exceeding a given ground motion Intensity Measure such as PGA (Peak Ground Acceleration), PSA (Pseudo Spectral Acceleration) etc. For this purpose, it is necessary to describe occurrence rates of earthquakes and the distribution of their magnitudes. This is the step 2 in the **Figure 1**. The most popular distribution of magnitudes is the exponential distribution from the Gutenberg-Richter (GR) law. Numerous studies and applications showed that the GR distribution is a reasonable model for defining the distribution of magnitudes in the lower and moderate magnitudes ranges. However, it deviates from the log-linear model in the higher frequency range. For this reason and to account for finite energy of faults, the GR distribution is generally truncated at a maximum possible magnitude value m_{max}. The justification of the choice of m_{max} from physics or simple statistics is not straightforward. Concurrently, recent analyses showed that the maximum magnitude can have a major impact on the hazard curve when high return periods as required for safety analysis of NPP (20 000 years) are considered.

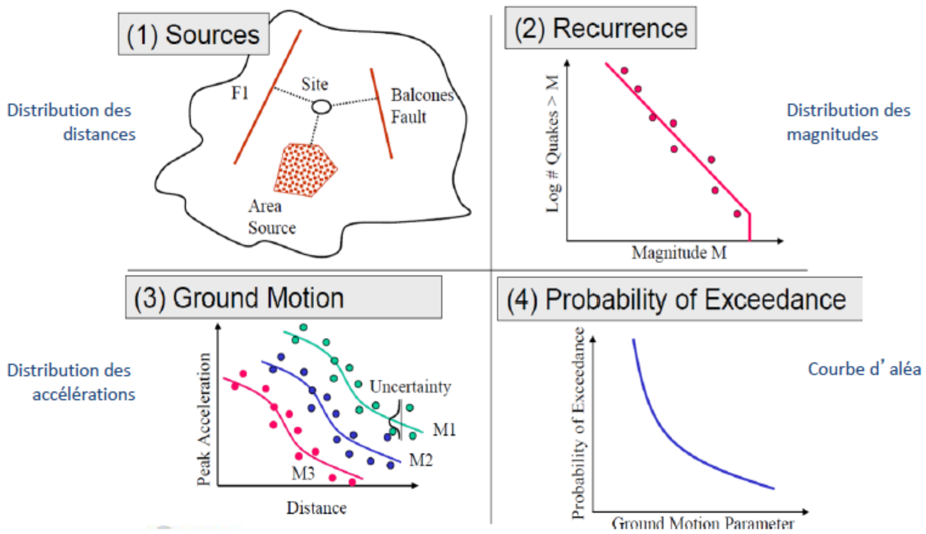

Figure 1: PSHA methodology.

The theory of statistics of extremes has been applied in engineering seismology since the early 'fifties by different authors such as Nordquist, 1945, Epstein & Lomnitz, 1966. The developments concern both the estimation of m_{max} of the truncated GR distribution and the direct estimation of the tails of the magnitude distribution by the generalized extreme value and Pareto distributions. Pisarenko et al., 2014, adopt the more general framework of the generalized extreme value distribution. The theory of extreme value statistics shows that the generalized extreme value distribution is the limit distribution of the maximum, of a series of independent random variables with same distribution under the condition of appropriate normalization. However, the scarcity of data in low seismicity regions can make it difficult to apply the latter methods. On the other hand, it is well known that the maximum likelihood estimator (MLE) of the magnitude used to truncate the GR law is biased (Kijko, 2004). The maximum likelihood estimate corresponds to the maximum of the likelihood function which is always equal to the highest observed magnitude m_{maxobs} in this case. As the number of observed earthquakes and thus the sample size n increases it becomes more and more likely that m_{maxobs} is the true m_{max} and the likelihood function gets more and more concentrated around this value. However, when increasing the sample size *n*, then the estimator converges to the "true" value but from below. Kijko developed a bias corrected maximum likelihood estimator to estimate m_{max}. The derivation of the correction term is however based on some simplifying assumptions.

This paper describes an improved approach for the estimation of the maximum magnitude in the truncated GR law by means of a Bayesian approach involving extreme value statistics and accounting for uncertainties based on Zentner et al (2020). The Bayesian updating approach adopted here allows for the combination of different sources of information, and to overcome the problem of bias of the simple maximum likelihood estimator (EPRI 1994, USNRC 2012).

The method to constructs the likelihood function based on the distribution of extremes of the truncated GR law and is an improvement of former developments by EPRI, see Johnston (1994), and further promoted by USNRC (2012). In the proposed method, only the completeness period of m_{maxobs} is required, so that there is no need to determine and use the exact completeness periods for magnitude bins of smaller events and to introduce the associated uncertainties. The development of the prior distribution of maximum magnitude relies on drawing analogies to tectonically comparable regions to increase the dataset for the development of generic distribution that can be updated for the considered region. This makes the approach easy to implement and to apply.

2. Method

2.1 Likelihood function derived from the distribution of extreme magnitudes.

Under the assumption of Poissonian occurrence, the cumulative density function (cf) function of maximum magnitudes over the period τ, reads:

$$G(m) = exp[-\lambda_0 \tau (1 - F_M(m))] \quad (1)$$

where λ_0 is the annual rate of earthquakes and $f_M(m)$ is the pdf of magnitudes. It is possible to derive the distribution of maxima accounting for the truncated GR law with upper bound m_{max} and lower bound m_{min}. In this case, we obtain the following expression for $m_{min} \leq \mathrm{m} \leq m_{max}$:

$$G(m) = \exp\left[-\lambda_0 \tau \left(\frac{\exp(-\beta m_{max}) - \exp(-\beta m)}{\exp(-\beta m_{max}) - \exp(-\beta m_{min})}\right)\right] \quad (2)$$

Where we have written λ_0 for the annual rate of earthquakes with magnitude larger than m_{min}.

The Bayesian updating allows for a robust and unbiased estimation. The likelihood functions are defined based on the extreme value distributions for Poissonian occurrences using the equations (2).

The parameters that define the likelihood function are the durations and the values of the maxima over this time interval. Numerical analyses showed that the result is the same when the catalogue is partitioned into equal intervals and the block maxima are used or when the maximum observed earthquake on the whole duration is considered. Since m_{maxobs} is the largest earthquake observed in the zone, we know that all other earthquakes observed over the period T of the catalogue are less or equal than this value. We use this information to write the likelihood function as the probability that the largest magnitude in the time interval T is less than m_{maxobs} (Zentner et al 2020):

$$L(obs|m_{max}) = G(m_{maxobs}|m_{max}) \quad (3)$$

This approach can be applied even if m_{maxobs} is outside considered completeness interval T. It allows considering the whole catalogue without considering the issue of completeness for smaller events, i.e. completeness other than for m_{maxobs}.

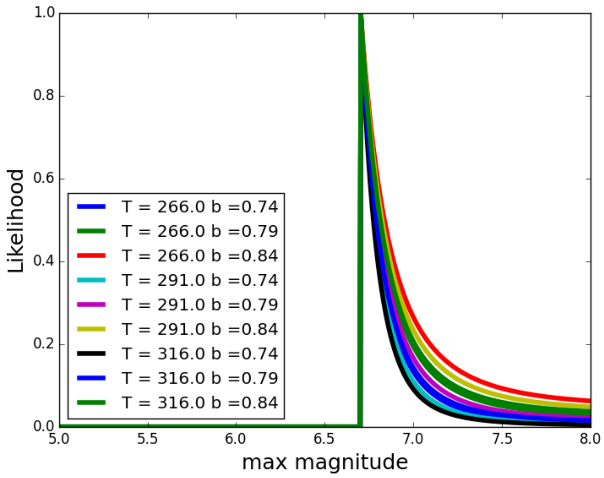

Figure 2: Illustration of likelihood functions obtained when accounting for uncertainty in b-value and completeness.

2.2 Magnitude and G-R parameter uncertainty

The impact of the uncertainty of the b-value and completeness on the likelihood function are highlighted in Figure 2. In addition, in Zentner et al (2020), the magnitude uncertainty has been analyzed based on numerical experiments with simulated catalogs. The standard-deviation (std) on the simulated magnitudes has been introduced with a dependance on the year observation. More precisely, the more ancient the observation, the higher the uncertainty, in agreement with values reported in FCAT 17 earthquake catalogue (Manchuel et al., 2017). It is found that the biais increases with true magnitude while the standard deviation is constant. This can be introduced in the updating procedure by means of an empirical distribution derived from the simulations. However, the biais depends not only on the maximum magnitude but also on the GR parameters and needs to be determined for the zone or region under study. Here we consider the mountainous zones in France with results given in the Figure 4 below.

Table 1 *Magnitude uncertainty: std for different years of observation*

	<1900	**<1950**	**<1975**	**today**
high	0.5	0.35	0.25	0.1

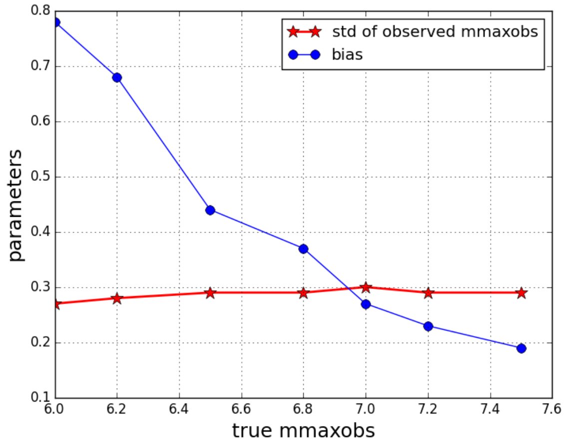

Figure 4: Mean and std of the difference between the observed m_{maxobs} and the true m_{maxobs} estimated from 5000 catalogues for French mountainous zones.

Rhoades (1996) pointed out that observed magnitudes have an associated bias that depends on the magnitude uncertainty. He derived the distribution of the biais and uncertainty with respect to the true magnitude m as:

$$f(m|\beta, x, \sigma) \sim N(m - (x - \sigma^2 \beta), \sigma^2)$$

Where x is the observed magnitude, σ is the associated standard deviation and β is the parameter from the G-R relation. Here we are interested in the maximum magnitude uncertainty, work is ongoing to check whether a (semi) analytical distribution can be derived for the maximum magnitude uncertainty.

2.3 Prior distribution

The implementation of the Bayesian approach requires the introduction of initial knowledge by means of the prior distribution. The prior distribution developed for France by Ameri et al 2015 are shown in Figure 5 below.

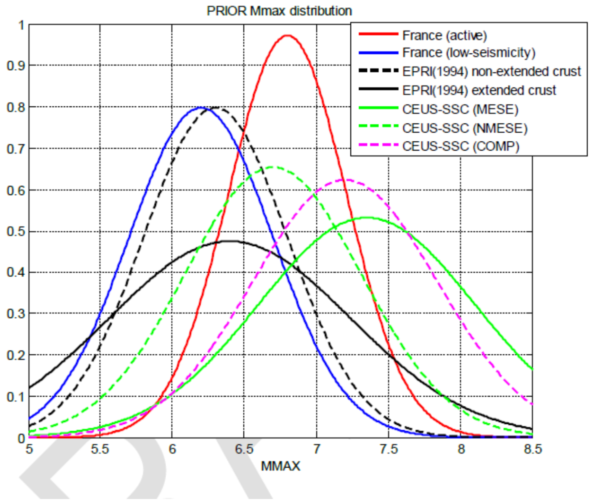

Figure 5: Priori distribution determined for France compared to former distributions from EPRI and CEUS, figure from Ameri et al (2015).

3. Application

In what follows, we apply the EPRI and the new Bayesian updating approach to French data from catalogue FCAT17 (Manchuel et al., 2017) considering magnitude uncertainty as explained above and the French prior distributions. Figure 6 shows the data from the catalogue for the considered French mountainous domains gathered in a macrozones of active regions (Ameri et al 2015). The largest observed magnitude in this macrozone is $m_{maxobs} = 6{,}7$, and it occurred within the completeness period T, highlighted by the vertical red bars.

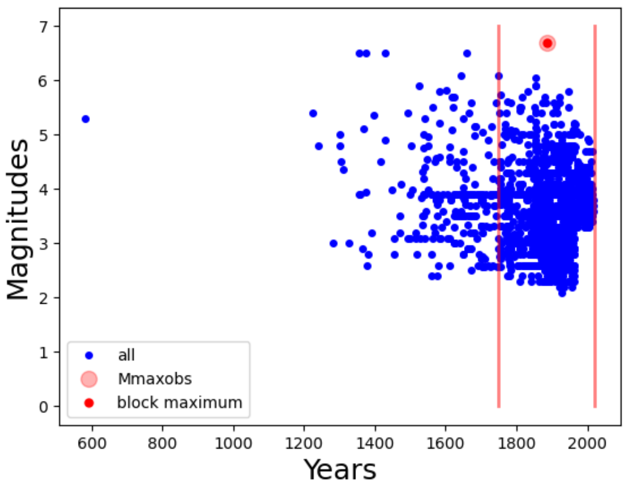

Figure 6: Data from earthquake catalogue for the considered French macrozone.

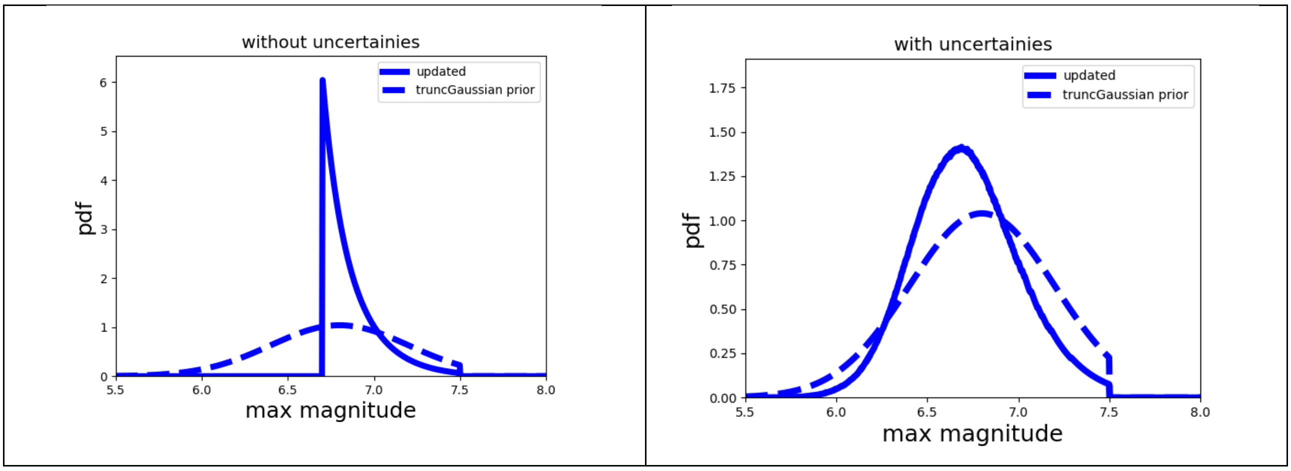

Figure 7: Prior and updated maximum magnitude distribution without accounting for magnitude and parameter uncertainty (left) and when considering uncertainty as described above (right)

Figure 7 shows the results of the Bayesian updating of the prior distributions of maximum magnitudes. Due to the uncertainty related to the observed m_{maxobs}, the likelihood function with uncertainty also allows for maximum magnitude values below the observed one, although higher magnitude values become more likely. The respective statistical values for the posterior distribution are given in table 2 for the truncated prior and considering prior without truncation. Results are very similar showing that the prior truncation does not have a significant impact on the result.

Table 2 : Statistics of the posterior distribution of m_{max} for truncated Gaussian prior distribution: mean, median and 5%, 95% fractiles. The results without truncation of the Gaussian prior a given in parenthesis.

Method	**Mean**	**Median**	**5% CI**	**95% CI**
This approach, with uncertainties	6.72 (6.73)	6.70 (6.71)	6.25 (6.25)	7. 22 (7.27)

4. Conclusions

We have proposed a new method that combines the distribution of extreme values of the truncated GR law with the Bayesian updating approach. In contrast to other existing methods, it allows for considering uncertainty on the GR parameters and the observed magnitudes. The simulated catalogues allowed for evaluating the bias (mean) and the std of the distribution of the difference between the true and the observed m_{maxobs}. The simulated results allowed to validate the approach and confirmed that the uncertainty on m_{maxobs} and the GR parameters can be integrated in the updating procedure in a straightforward way.

The proposed approach is easy implement since it requires only knowledge on the maximum magnitude and the associated completeness interval to update the distribution of maximum magnitudes.

5. REFERENCES

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