FRAMEWORK TO INTRODUCE SITE RESPONSE IN SEISMIC RISK ASSESSMENT AND APPLICATION TO METIS CASE STUDY

V. Alves-Fernandes[1], M. Korres[1], I. Zentner[1]

[1] EDF R&D, Lab Paris-Saclay, France

***Abstract**: The goal of this work is to develop a comprehensive strategy for defining ground motion and soil columns for SSI and floor response. A first step consists in the evaluation whether 1D or multidimensional (2D, 3D) site response analysis are required for the site under study. In this work, we present and consider the METIS case study site located in central Italy, which is subjected to moderate seismicity such as often encountered in regions with nuclear installations in European countries. One major bottleneck in the site response analysis is the consideration of uncertainty. The 2D and in particular 3D analysis come at high computational cost which reduces the number of affordable analysis and the feasibility of comprehensive sensitivity and uncertainty studies. The 1D computations run very fast, however, whenever applicable (that is absence of pronounced 3D effects), such analysis requires the consideration of uncertainty due to the simplifying assumptions on lateral variability and incoming wavefield. Therefore, we develop an approach to define consistent (linear) equivalent 1D columns reflecting the observed variability and attenuation expected in the considered site. We perform complementary 2D analysis to analyse the impact of 2D wave propagation and lateral variability.*

1. INTRODUCTION

The seismic risk assessment of nuclear power plants includes seismic hazard, fragility, and system assessment. The strategy developed in METIS project is to develop seismic hazard on rock conditions and to consider the impact of the surrounding soil and site conditions as part of the structural analysis. This approach has the advantages that it allows developing site response only for the scenarios of interest for the engineer. This means that this strategy requires much less analysis than what would be required for introducing detailed site response in hazard curves and avoids double counting of related uncertainties.

In the current nuclear practice and most regulations, seismic hazard is defined at the free-field where site response is accounted for in a simplified way through Ground Motion Models (GMM) and then again modelled with more detail in the structural response and SSI modelling step. The latter generally implies double counting and possibly bias due to conservative assumptions at the interface between hazard and structural response. This can be overcome by developing a unified approach where seismic hazard is defined on rock and bedrock conditions while site response is introduced in the structural modelling step. This is illustrated in Figure 1. The soil-profile and soil surface ground motion pairs are then handed over for the computation of structural floor response including Soil Structure Interaction.

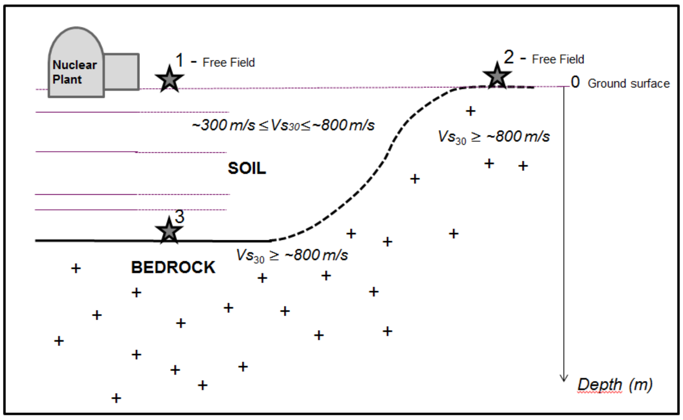

Figure 1. Schematic view of the site configuration and the definition of the « reference or control point » on bedrock (3) and free field (1,2). Figure from (Berge-Thierry, et al., 2017).

The approach presented here is fully in line with findings from SINAPS@ project (Berge-Thierry, et al., 2017) and (Laurendeau, et al., 2017) where a control point at the bedrock level and elaboration of specific GMM for rock site condition is proposed. Defining the hazard for rock and bedrock conditions requires the availability of appropriate rock GMM. In this respect it is interesting to acknowledge the recent efforts to develop rock GMM for the computation of hazard on rock and bedrock conditions. For example, Shible et al (2023) recently developed ground motion for rock sites from deconvolved ground-motion models using site response from generalized inversion techniques. Concurrently, recent GMM developers (Al Atik & Abrahamson 2021) provide compatible generic soil profiles that would allow to determine the reference bedrock motion from surface motion. Obviously, despite increasing effort in site characterization, the soil conditions and the parameters governing the seismic wave propagation are not perfectly known. Therefore, it is paramount to carefully assess and propagate uncertainties in the framework of the 1D soil columns analysis.

Given hazard and ground motion at rock level, a first step consists in the evaluation whether 1D or multidimensional site response analysis are required for the site under study. This depends on the configuration of the site, such as only horizontal stratification of the soil or the presence of prominent geological features such as a basin, layer folding and topography that can have a major impact on the wave field. This is not further discussed here but the reader is referred to Korres et al (2024).

In what follows we describe first the study case from METIS project, and then we introduce approaches and results for 1D and 2D site response analysis. Based on simple inspections the site was judged suitable for 1D analysis. For the 1D site response we conduct several comparative studies (soil uncertainty, column heights) and we assess the relative impact on PGA variability from the selected input motions and soil uncertainty. Then we provide some preliminary comparison to 2D analysis to assess the impact of the not perfectly horizontal soil layering.

2. SITE DESCRIPTION

The METIS case study site is located in Tuscany, Italy, facing the Tyrrhenian sea. Geotechnical boreholes conducted at the site showed 4 geotechnical layers, whose elastic characteristics are summarized in **Table 1**. Numbering starts at 2 as the 2m vegetation covering is neglected in this study.

The bedrock layer could not be directly identified neither from geotechnical borehole characterization nor geophysical cross-holes (100m depth maximum). Geophysical down-hole measurements (**Figure 1**) showed a nearly flat interface between the first geotechnical layers (horizon A corresponding to the interface between layers 2 and 3 and horizon B the interface between layers 4 and 5), but a variable bedrock interface depth along the site, with a maximum depth around 500-600m. From the geological point of view, bedrock corresponds to a flysch rock, for which Vs values have also to be defined. A reasonable hypothesis considered in this study is to consider Vs for the bedrock of 1000m/s, which is also in accordance with the rock Vs considered for the PSHA established for the METIS case (Chartier and Rood, 2024).

From this hypothesis, Vs values for Layer 5 are determined as a power law $Vs(z) = V_0(1 + z)^n$, with Vs(78m)=417m/s and Vs(500m)=1000m/s.

Table 1 – Site geotechnical properties.

Layer		Depth (m)	Soil nature	Vs (m/s)	Poisson ratio ()	Mass density $\rho (t/m^3)$
Layer 2	2-1	0-5	Sand and gravel	509	0.4	1.75
	2-2	5-10	Silt and sand	390	0.475	1.70
	2-3	10-15	Sand and gravel	405	0.455	1.80
	2-4	15-18	Sandy silty and sandy clay	605	0.425	1.85
	2-5	18-25	Gravelly sand	590	0.45	1.90
	2-6	25-30	Silty sand and silty clay	489	0.455	1.95
	2-7	30-33	Sand and sandstone	805	0.45	1.95
Layer 3		33-64	Silty clay	380	0.45	2.00
Layer 4		64-78	Silty sand and sandy silt	430	0.45	2.05
Layer 5		from 78	Silty clay	Vs(z)	0.455	2.00

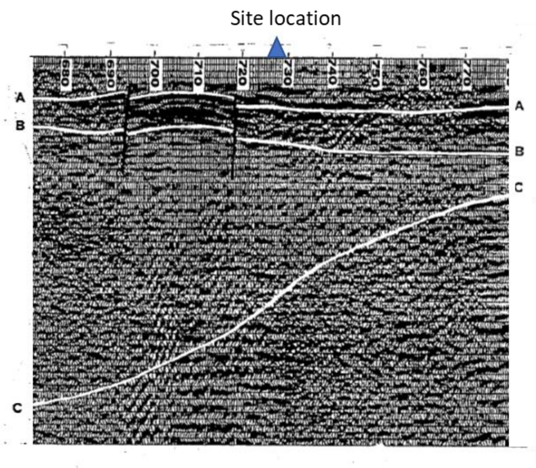

Figure 1 – Synthetic view of one down-hole measurement performed at the site.

The dependence of normalized secant shear modulus (G/G_{max}) and damping (D) to shear strains were partially characterized by resonant column tests on soil specimens from layers 2 and 3 (**Figure 3**, **Figure 4**), as maximum shear strains were around 3.10^{-4} and soil is expected to be subjected to larger shear strains during extreme shaking. Therefore, models for both $G/G_{max}(\gamma)$ and $D(\gamma)$ are considered to cover a larger shear strain range, as follows:

- Normalized secant shear modulus: adjustment from a simple hyperbolic model: $\frac{G}{G_{max}} = \frac{1}{1+\left(\frac{\gamma}{\gamma_r}\right)^a}$
- Damping: adjustment for both Darandelli (2001) and Zhang et al. (2004) models, and the mean damping curve from these two models is considered.

As no data was provided for layers 4 and 5, this study considers the same normalized secant modulus and damping for layers 2/4 and 3/5, based on similarities on soil nature, although these layers are not expected to be subjected to large non linearities.

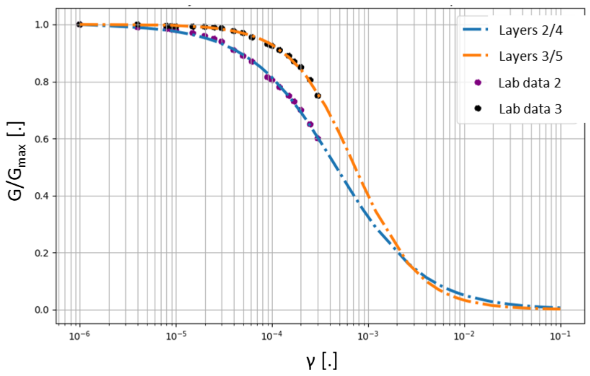

Figure 2 – Normalized secant modulus curves.

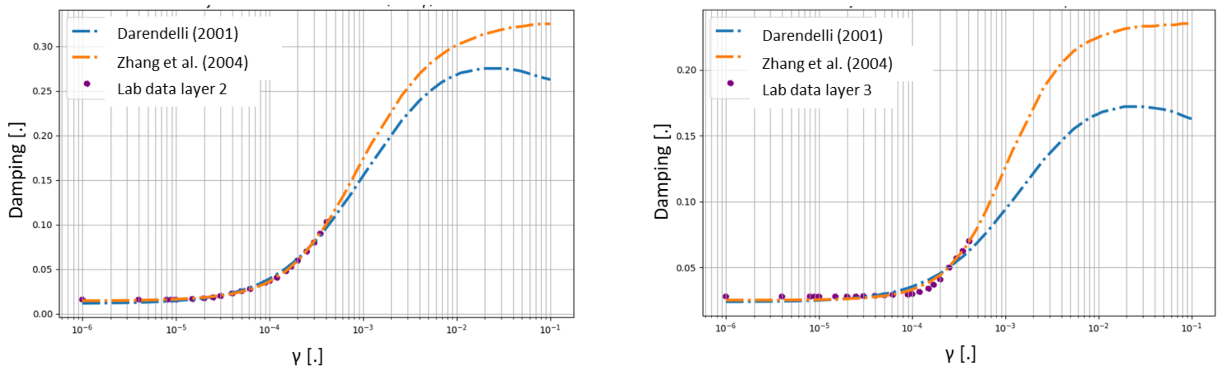

Figure 3 – Damping curves.

3. 1D MODELING

In this section we describe and assess an approach that allows for the consideration of uncertainty, variability and attenuation in equivalent linear (EQL) response. The equivalent linear (EQL) approach is based on an iterative procedure of linear elastic simulations in frequency domain, for which soil dynamic properties (shear modulus and hysteretic damping) is updated at each iteration and for each soil layer based on the 65% of the maximum shear strain obtained during the shaking.

We quantify and propagate uncertainty and perform sensitivity analysis to define a reduced set of 1D columns that would: (i) well represent the mean and standard deviation of soil surface ground motion variability and (ii) be easily implemented for soil-structure interaction studies.

3.1 Overall strategy and uncertainty propagation

The prediction of the one-dimensional (1D) seismic site response consists of vertically propagating seismic shear waves through horizontally layered and laterally infinite homogeneous soil until the soil surface.

The following modeling strategy is considered in this study to obtain soil surface ground motion:

- Deconvolution of rock motions selected at the free-field to the bedrock depth
- Convolution of the obtained time-histories through a 1D soil column

The deconvolution considered a homogeneous rock with Vs 1000m/s and 0.1% damping (following Vs/10 rule of thumb for quality factor estimation). Convolution considers a bedrock at 200/400m depth, in accordance with obtained data (**Figure 1**).

Here, both the variability of input ground motion on rock as well as the uncertainty related to site data are considered and propagated. The Toro (1995) model was extensively used in the past for 1D site response analysis (Li & Assimaki 2010, Rodriguez-Marek et al 2014). Toro (1995) used a large Vs database to construct generic and site-specific Vs randomization models, provided a generic site classification and a base case Vs profile. However, this approach is not perfectly suited for best-estimate and site-specific applications, when borehole and geophysical site data is available. Also, it does not necessarily guarantee the experimental site signatures and can lead to excessive variability of shear wave velocity profiles and an overestimation of the expected variability in site response estimates (Passeri et al 2020).

We adopt a generic framework for consideration of uncertainty and propagate uncertainty Latin Hypercube sampling, see **Figure 4** below.

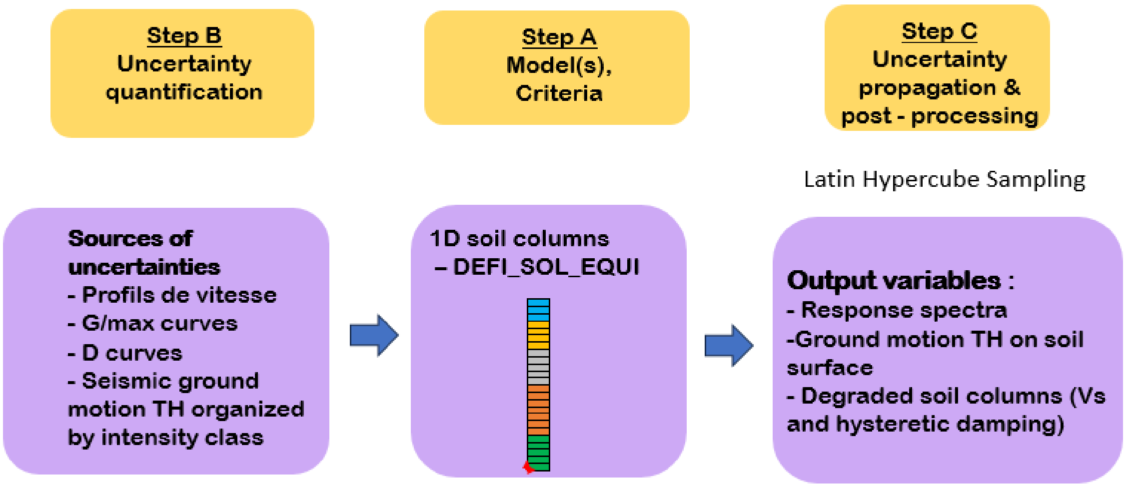

Figure 4 – Uncertainty propagation scheme.

A lognormal distribution is assumed for the sampling of the damping (D) curves while a truncated Gaussian distribution was chosen for the G/G_{max} curves due to their specific properties. A negative correlation of -0.5 is introduced to account for the increasing damping with decreasing modulus. **Figure 5** shows the standard deviation from Darendeli (2001) for generic and different specific soil types. We adopted the lower specific values for layer 3,4 and 5 of our soil profiles.

The correlated Vs values are also modelled by lognormal distribution where the correlation depends on the vicinity of the layers, i.e. adjacent layers have higher Vs correlation than more distant ones. Uncertainty in the layer heights is not accounted for. **Figure 6** shows a sample of 100 soil profiles together with the median and +-1 sigma intervals.

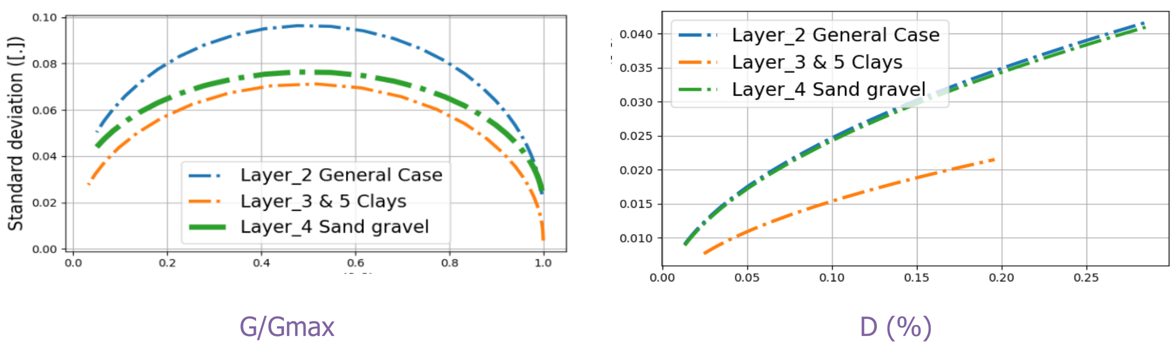

Figure 5 – Generic and specific soil class standard deviation from (Darendeli, 2001), layer numbers refer to METIS case study.

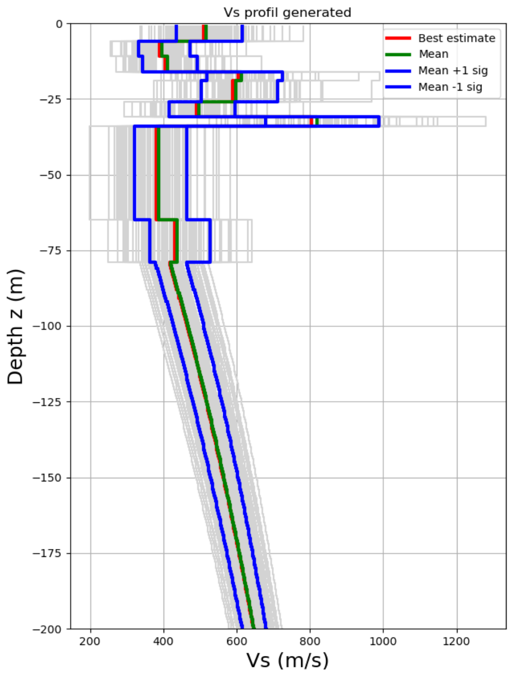

Figure 6 – Set of N=100 probabilistic soil columns for H=200m hypothesis.

In our example applications we identified a sample size of N=100 to well represent soil uncertainty but this must be assessed for more examples to deduce general conclusions. For the uncertainty propagation, we pair each of the 100 soil columns with one time history from the set. Since 100 soil columns for 25 time histories are available (for each horizontal direction), each time history is used 4 times. The results for this LHS-type design are compared to the full factorial design where each time history is paired with each soil column. The results confirmed the acceptability of the reduced design.

3.2 Ground motion

For our application, we work with the set of time histories selected based on the Conditional Spectra approach by the METIS project partner IUSS. For the CS- approach we considered 5 intensity measure levels (IML) equivalent to 5 return periods: 2500, 5 000, 10 000, 20 000, 50 000 years. The target uniform hazard spectra (UHS) and conditional spectra (CS – median and +- 1std) are shown in **Figure 7**. The PGA is chosen as the conditioning IM. The hazard and the resulting target spectra have been computed with openquake for METIS case study PSHA (see Chartier & Rood 2024 for more information on the hazard study).

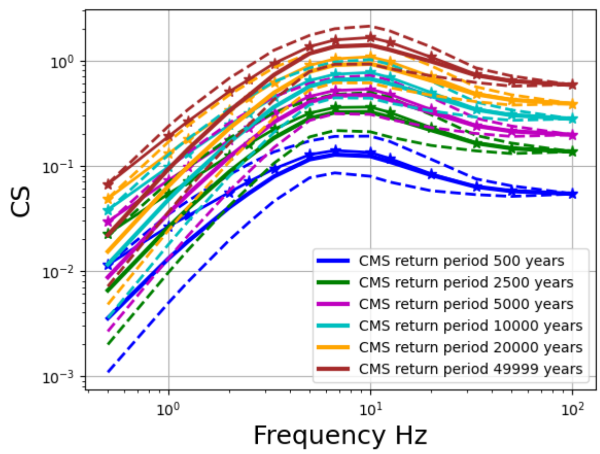

Figure 7 – Seismic ground motions.

3.3 Results and discussion

The proposed methodology is first applied by considering best-estimate soil properties for soil columns of 200m and 400m depth and the selected 25 time histories at for one horizontal direction and for each IML. For return period of 2500 years, median and 15%/85% quantile results are equivalent regardless the soil column depth, whereas for the highest considered IML (return period of 50000 years), slight differences are observed (**Figure 8**) and linked to nonlinearities occurring at the deepest soil layer (Silty clay).

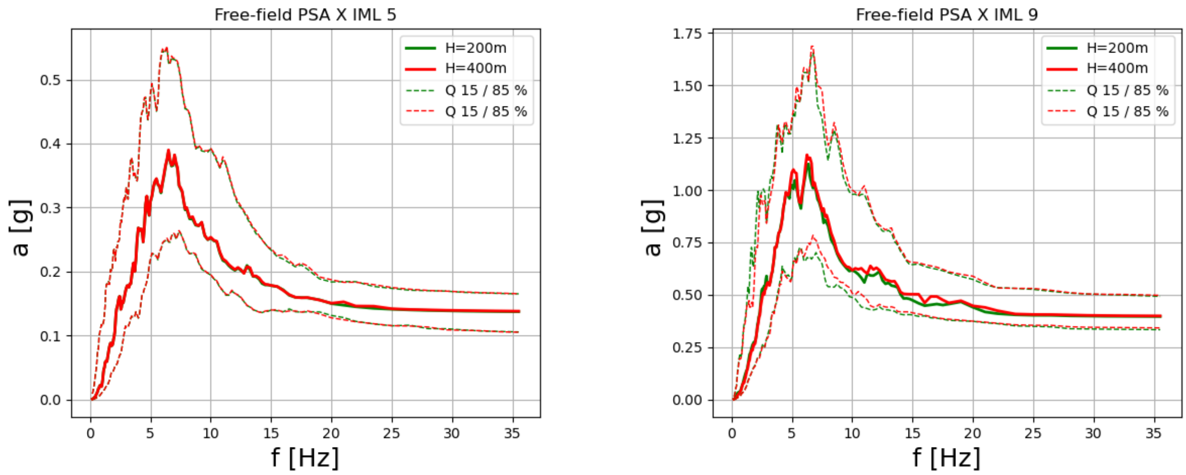

Figure 8 – Pseudo-acceleration for return period of 2500 (left) and 50000 (right) years for soil column hypothesis of 200m or 400m depth. Solid line: median; dashed line: 15% and 85% quantiles.

From the numerical simulations performed, the introduction of uncertainties in the soil column (Vs profile and nonlinear properties) directly impacts the bedrock to surface transfer function (**Figure 9**) as: (i) smaller resonance pics for the median TF values are observed and (ii) higher TF variability in comparison to the best estimate profile only, although differences reduce when considering high return periods. However, median and 15%/85% quantiles for the spectral accelerations at the surface are less impacted by the variability at the soil column (**Figure 10**). Slight lower median values and consistently lower 15% quantile are obtained for return period of 50000 years, as higher nonlinearities are attained for some simulations, leading to lower spectral accelerations.

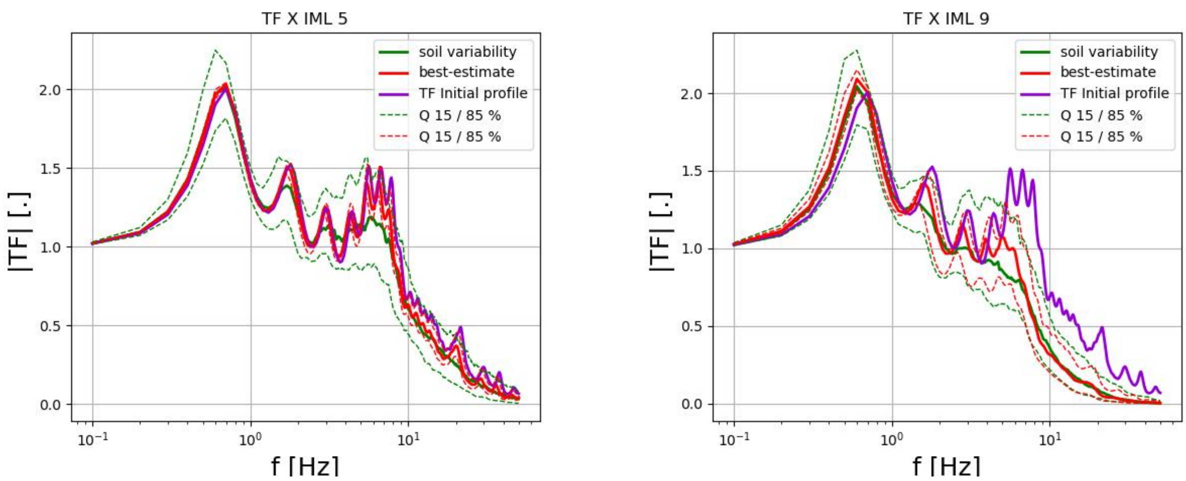

Figure 9 – Bedrock to surface transfer function for return period of 2500 (left) and 50000 (right) years. Solid line: median; dashed line: 15% and 85% quantiles.

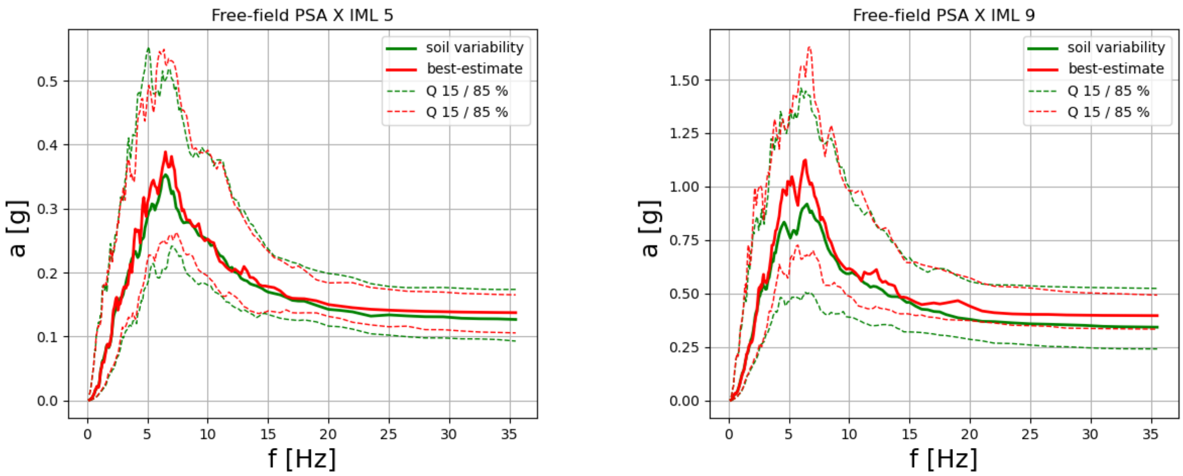

Figure 10 – Pseudo-acceleration for return period of 2500 (left) and 50000 (right) years when considering uncertainties in the soil column. Solid line: median; dashed line: 15% and 85% quantiles.

The relative impact of introducing uncertainties in the soil column compared to the variability of input time histories only is analysed by comparing the PGA coefficient of variation (COV) mean value for both horizontal directions (**Figure 11**). We obtain consistently higher COV values for the case including variability in the soil, although the larger part of the variability in PGA may be attributed to the variability in soil column response to the selected input time histories rather than the randomized soil profiles.

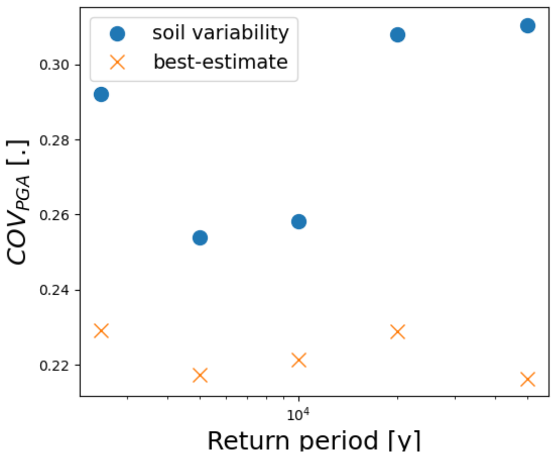

Figure 11 – Coefficient of variation (COV) for PGA at different return periods.

4. 2D MODELING

In this section we describe and assess an approach that allows for the consideration of multi-dimensional site effect analysis using the so called Free-Field Boundary Condition (FFBC). It is most common in practical applications that information concerning the seismicity of the region of interest are constrained to a PSHA analysis performed for the reference bedrock, while information concerning the form of the complete wave field is quite scarce. In this context, it is common practice to evaluate site amplification using a plane wave excitation of vertical incidence generally applied at the bottom of the computational domain, while the motion along the lateral sides in not known beforehand. A simple solution in order to amend for this uncertainty of the lateral ground motion, initially proposed by (Zienkiewicz, et al., 1989), is the FFBC, where the excitation imposed on the lateral sides is the one obtained from the wave propagation in a 1D soil column as schematically represented in **Figure 12.** Schematic representation of the free-field boundary condition (FFBC).**Figure 12**.

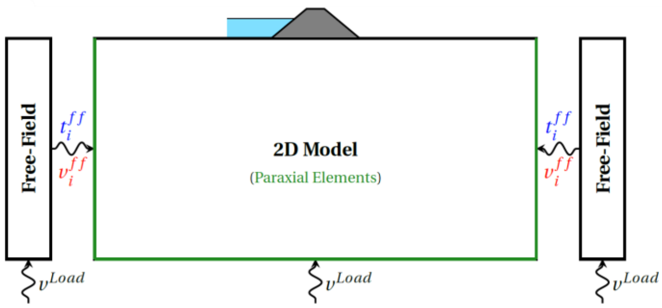

Figure 12. Schematic representation of the free-field boundary condition (FFBC).

The definition of the dynamic excitation in code_aster is ensured via the use of paraxial elements. The traction vector t_i^{ff} and the incident velocity field v_i^{ff} are obtained from the 1D soil column wave propagation. These are the necessary fields needed to correctly construct the free-field excitation on the lateral boundary (Korres et al. 2022).

It is worth noticing that the 1D columns can be solved prior the construction of the dynamic excitation which is then transferred to the lateral boundary for the 2D dynamic analysis. This implies that no interaction takes place between the 1D column and the 2D profile as information is transferred unidirectionally from the 1D column to the 2D domain. This implies that the wave field generated in the 2D model does not affect the free-field ground motion of the 1D propagation, a bold assumption that might be quite unrealistic depending on the complexity of the 2D analysis. Nevertheless, it can be justified in certain cases if the columns are placed at some distance from the central region of the model.

3.4 Overall strategy and assumptions for the 2D analysis

The aforementioned methodology is applied in the 2D soil profile of the METIS case-study (**Figure 1**) by considering the best-estimate soil properties equivalent to the soil column with H=200m. However, contrary to the 1D analysis, no material damping is considered in this case. The spatial variation of the Young's modulus for the 2D soil profile of **Figure 1** and the best-estimate properties is presented in **Figure 13** (left).

Provided that the efficiency of the FFBC approach is highly influenced by the distance between the geology of interest and the lateral boundary of the model, a first comparison is performed using a simplified Ricker excitation so as to converge on the appropriate size for the 2D model. For this purpose, a hypothesis is made for the lateral extension of 2D soil profile (**Figure 1**) assuming horizontal layer stratification in both sides of the model. The latter was mainly motivated from the lack of further information allowing to extend the 2D profile more appropriately as well as to the fact that the site is located close to the seacoast and thus the horizontal extension of the model remains plausible. Consequently, three model sizes are examined as presented in **Figure 13**, i) *Small* model, corresponding to the known 2D profile, and ii) *Medium* and *Large* model, allowing to evaluate the convergence of the numerical solution at the site of interest.

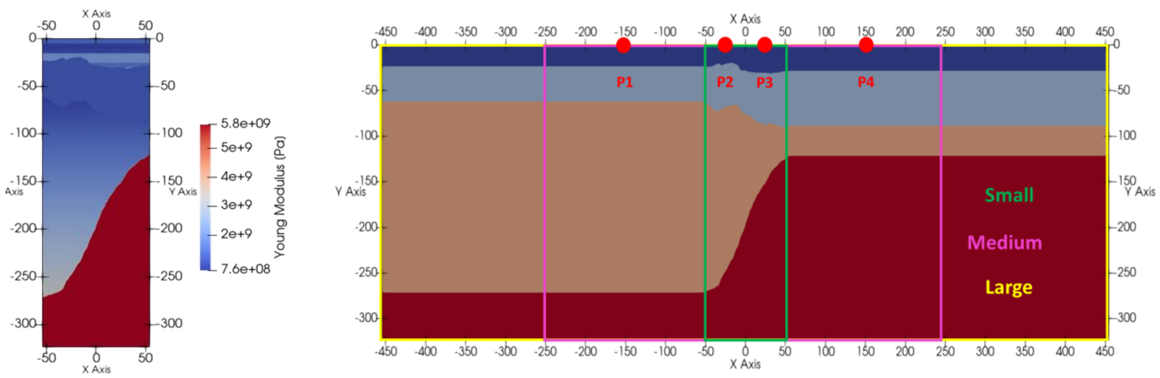

Figure 13. Spatial variability of the Young's modulus for the small model (left), and the three different sizes of the 2D soil profile (right).

Once the 2D model size is fixed, a comparison is performed between the 1D and 2D site response, using the previously described ground motion database proposed for the METIS case study. The purpose of this analysis is to evaluate the "1D character" of the site of interest and evaluate the spatial variability of the ground motion deriving from the 2D analysis.

3.5 Results and discussion

The influence of the size of the model is examined at first. Three models are examined adopting the same boundary conditions: plane wave excitation with vertical incidence imposed at the base of the model and the free-field motion obtained from the 1D soil column imposed at the lateral side. Numerical results for the three model sizes are presented in **Figure 14**, and for a point located at the site of interest (surface and the center of the model). It is worth noticing that the *Small* model (green line in **Figure 14**) provides a different response compared to the other two model sizes for the convergence in terms of site response. Two main reasons can be the source for this important difference in terms of site response:

- The free-field motion imposed at the boundary steams from an 1D propagation on a horizontally stratified soil column which is incompatible with the wave front generated the 2D geology. In that sense the proximity of the boundary to the 2D geological profile plays a negative role on the global response even more due to the lack of material attenuation in the present study.
- As illustrated in the acceleration wave field presented in **Figure 15**, the 2D soil profile modifies locally the vertical incidence of the wave front leading to an inclined wavefront following the geological profile. Paraxial elements are known to perform better for angles of incident waves that remain close to 90^0 (Modaressi et al. 1994). In this context, different angles of incidence may decrease their absorption capacity and lead to spurious reflections on the domain of interest. This phenomenon is less present as the boundary remains farther from the complex geology.

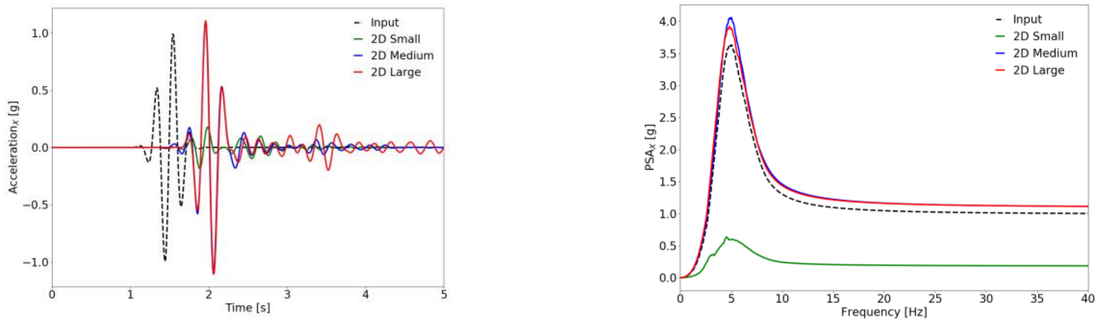

Figure 14. Horizontal component of the site response and a simplified Ricker excitation: acceleration time history (left), and 5% damped spectral acceleration (right).

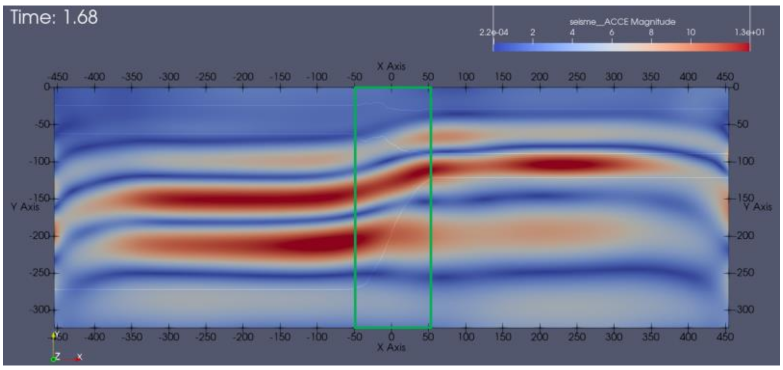

Figure 15. Acceleration field at 1.68s. The green rectangle at the center represents the size of the small model.

Provided the small differences between the *Medium* and *Large* size models, the *Medium* size model is chosen so as to examine the 2D site response compared to the 1D soil analysis.

Since the 2D analysis are conducted as linear elastic with no material damping, numerical results are presented here only for the return period of 2500 years and all 25 acceleration time-histories and along the x component. Comparison between the 1D and the 2D (point at the center and surface) site response is presented on the left side of **Figure 16**, in terms of mean value (solid line) and standard deviation (dashed line) for the 25 seismic signals. According to this **Figure 16**, site amplification is similar between the 1D and the 2D case and thus the assumption of 1D propagation is justified for the case of interest, although a larger frequency content is obtained for the 2D analysis compared to 1D.

Nevertheless, it is worth noticing that contrary to the 1D analysis, the 2D simulation also accounts for the spatial variability of the seismic ground motion. At the right side of **Figure 16**, we compare the results for one of the seismic signals and different stations at the surface of the 2D profile (see also right side of **Figure 13**), where we observe that the response is different depending on the location of the point at the surface (distance of 300m between point 1 and 4). The latter is an important element that needs to be considered as the variability of the ground motion may lead to more complex excitation on the level of structure of interest compared to a simplified excitation steaming from a plane wave solution and 1D soil column hypothesis.

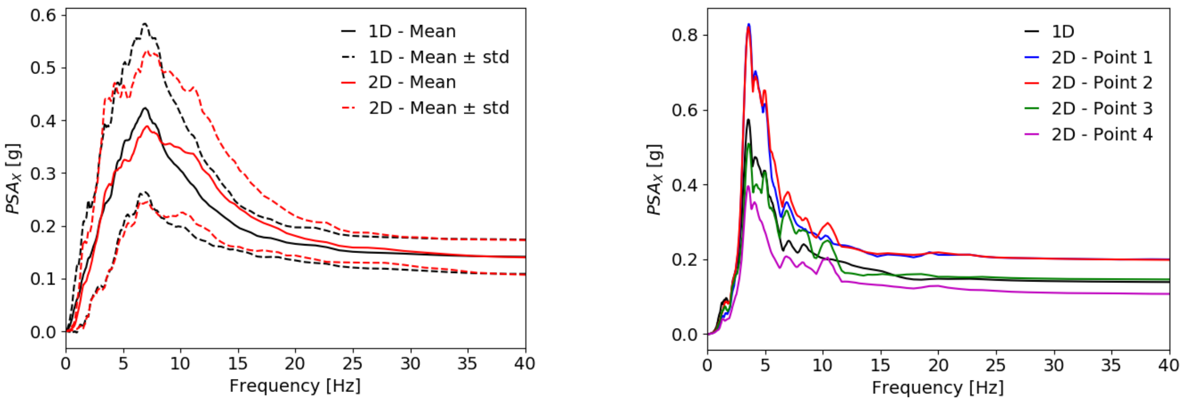

Figure 16. Horizontal component of the 5% damped spectral acceleration: Comparison 1D vs 2D (left), Spatial variability of the seismic ground motion (right).

5. CONCLUSIONS AND PERSPECTIVES

In this work a state-of-the-art methodology for uncertainty quantification and propagation for site amplification estimation is applied for the METIS case. The interpretation of the geophysical and geotechnical data allowed for determining soil parameters and a geometrical description of the site, both under 1D and 2D hypothesis.

The performed 1D analysis provided insights on how rock-to-surface transfer function and free-field PGA variability are impacted by uncertainties in the soil column and material properties and the variability of hazard consistent ground motions. The data produced by 1D modeling (free field ground motions and linear equivalent soil columns) is expected to nourish the SSI numerical simulations conducted in METIS project. These results could be further confronted to nonlinear soil predictions, to give higher confidence on linear equivalent approach as part of METIS strategy to match site effects estimation and SSI modeling.

The 2D analysis presented here manipulates the free-field boundary condition approach for a more accurate consideration of the ground motion at the lateral sides of the 2D model. The analysis aims to illustrate the importance of the size of the model when complex geological profiles are numerically simulated. Their presence alters the wave-front leading to more complex propagation pattern that the traditional plane wave of vertical incidence. An illustration is provided for a simplified excitation in order to converge to a solution at the level of the site of interest. For a fixed size model, 2D site response is compared with the 1D analysis where a similar response is observed in both cases validating the hypothesis of the 1D site response in terms of site amplification at the center of the model. The spatial variability of the seismic ground motion is also examined for several points at the surface of the 2D profile, where we observe that even for points close to the center of the model, site response is altered due to the presence of the 2D profile. The influence of the latter in structural response is subject to the characteristics of the structure and its interaction with the surrounding soil and it should be evaluated to rule out (or not) the influence of multidimensional site effects.

6. REFERENCES

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